Matlab Tools for Computing Canonical Structure Information

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The Settings

Consider a matrix pencil

$$G - \lambda H, \qquad \lambda \in \mathbb{C}$$

where $G, H \in \mathbb{C}^{m\times n}$

Goal

Robust Matlab software for computing and analyzing canonical structure information

Code contributions also by A. Dmytryshyn and P. Johansson

Canonical Structure Information

A canonical form (CF) reveals the canonical structure information from which the system characteristics are deduced

- Matrices A
- Matrix pencils $G - \lambda H$
- System pencils $S - \lambda T$
- Matrix polynomials $P(\lambda) = \lambda^d A_d + \ldots + \lambda A_1 + A_0$

Canonical Structure Information

A canonical form (CF) reveals the canonical structure information from which the system characteristics are deduced

- Matrices A – *Jordan CF*
- Matrix pencils $G - \lambda H$ – *Kronecker CF*
- System pencils $S - \lambda T$ – *Brunovsky-type CF*
- Matrix polynomials $P(\lambda)$ – *Smith CF*

A canonical form is the simplest or most symmetrical form a matrix or matrix pencil can be reduced to

Jordan Canonical Form

The *Jordan canonical form* (JCF) reveals the fine eigenstructure of a square matrix A:

$$SAS^{-1} = J, \quad \text{where } S \text{ is non-singular}$$

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J is a *block-diagonal matrix* where each block is a $k \times k$ *Jordan block* (a regular block) for an eigenvalue μ of A:

$$J_k(\mu) = \begin{bmatrix} \mu & 1 & & \\ & \mu & \ddots & \\ & & \ddots & 1 \\ & & & \mu \end{bmatrix}$$

Geometric multiplicity – Number of Jordan block for μ
Algebraic multiplicity – Total size of all Jordan blocks for μ

Kronecker Canonical Form

For *non-square* matrices and matrix pencils $G - \lambda H$, the canonical form may also consist of *singular blocks*

The *Kronecker canonical form* reveals the fine eigenstructure of a matrix pencil

Kronecker Canonical Form

Any matrix pencil $G - \lambda H$ can be transformed into *Kronecker canonical form* (KCF) using equivalence transformations (U and V non-singular):

$$U(G - \lambda H)V^{-1} =$$
$$\mathrm{diag}(L_{\epsilon_1}, \ldots, L_{\epsilon_p}, J(\mu_1) - \lambda I, \ldots, J(\mu_t) - \lambda I, N_{h_1^{(\infty)}}, \ldots, N_{h_g^{(\infty)}}, L_{\eta_1}^T, \ldots, L_{\eta_q}^T)$$

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Singular part:

- $L_{\epsilon_1}, \ldots, L_{\epsilon_p}$ – Right singular blocks
- $L^T_{\eta_1}, \ldots, L^T_{\eta_q}$ – Left singular blocks

$$L_k = \begin{bmatrix} -\lambda & 1 & & \\ & \ddots & \ddots & \\ & & -\lambda & 1 \end{bmatrix}$$

of size $k \times (k+1)$

Kronecker Canonical Form

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Singular part:

- $L_{\epsilon_1}, \ldots, L_{\epsilon_p}$ – Right singular bloc
- $L_{\eta_1}^T, \ldots, L_{\eta_q}^T$ – Left singular block

$$J_k(\mu_i) - \lambda I = \begin{bmatrix} \mu_i - \lambda & 1 & & \\ & \ddots & \ddots & \\ & & \ddots & 1 \\ & & & \mu_i - \lambda \end{bmatrix}$$

Regular part:

- $J(\mu_1), \ldots, J(\mu_t)$ – Each $J(\mu_i)$ is block-diagonal with *Jordan* blocks corresponding to the finite eigenvalue μ_i
- $N_{h_1^{(\infty)}}, \ldots, N_{h_g^{(\infty)}}$ – Each $N_k = I - \lambda J_k(0)$ and corresponds to the infinite eigenvalue

STAIRCASE FORMS

Using orthogonal or unitary matrices and backward stable algorithms:

Matrices The Jordan-Schur form [Kublanovskaya'66]

Matrix pencils A generalized Schur-staircase form [Van Dooren'79], e.g. the *Guptri staircase form* [Demmel-Kågström'93]

Staircase Forms

Using orthogonal or unitary matrices and backward stable algorithms:

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Matrix pencils A generalized Schur-staircase form [Van Dooren'79], e.g. the *Guptri staircase form* [Demmel-Kågström'93]

Computing the canonical structure information is an **ill-posed problem**!

THE GUPTRI STAIRCASE FORM

GUPTRI – Generalized UPper TRIangular form
[Demmel-Kågström'93]

$$P^H(G - \lambda H)Q = \begin{bmatrix} A_r - \lambda B_r & * & * \\ 0 & A_{reg} - \lambda B_{reg} & * \\ 0 & 0 & A_l - \lambda B_l \end{bmatrix}$$

THE GUPTRI STAIRCASE FORM

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$$P^H(G - \lambda H)Q = \begin{bmatrix} A_r - \lambda B_r & * & * \\ 0 & A_{reg} - \lambda B_{reg} & * \\ 0 & 0 & A_l - \lambda B_l \end{bmatrix}$$

where P and Q are unitary matrices

$A_r - \lambda B_r$ – Right singular part; L blocks
$A_l - \lambda B_l$ – Left singular part; L^T blocks

$A_r - \lambda B_r$ and $A_l - \lambda B_l$ are block-upper triangular

THE GUPTRI STAIRCASE FROM - REGULAR PART

$$A_{reg} - \lambda B_{reg} = \begin{bmatrix} A_z & * & * \\ 0 & A_f & * \\ 0 & 0 & A_i \end{bmatrix} - \lambda \begin{bmatrix} B_z & * & * \\ 0 & B_f & * \\ 0 & 0 & B_i \end{bmatrix}$$

where $A_{reg} - \lambda B_{reg}$ is upper triangular and

$A_z - \lambda B_z$ – The zero eigenvalue $\mu = 0$

$A_f - \lambda B_f$ – Finite non-zero eigenvalues μ

$A_i - \lambda B_i$ – The infinite eigenvalue $\mu = \infty$

The GUPTRI Staircase From - Regular Part

$$A_{reg} - \lambda B_{reg} = \begin{bmatrix} A_z & * & * \\ 0 & A_f & * \\ 0 & 0 & A_i \end{bmatrix} - \lambda \begin{bmatrix} B_z & * & * \\ 0 & B_f & * \\ 0 & 0 & B_i \end{bmatrix}$$

where $A_{reg} - \lambda B_{reg}$ is upper triangular and

$A_z - \lambda B_z$ – The zero eigenvalue $\mu = 0$

- $a_i = 0$ and $b_i \neq 0$

$A_f - \lambda B_f$ – Finite non-zero eigenvalues μ

- $\mu_i = a_i/b_i$ where $a_i \neq 0$ and $b_i \neq 0$

$A_i - \lambda B_i$ – The infinite eigenvalue $\mu = \infty$

- $a_i \neq 0$ and $b_i = 0$

The Guptri Algorithm

Seven steps in total:

1. Compute the RZ-staircase form (RZ-algorithm):
   Right singular structures and zero eigenvalues in $A_{rz} - \lambda B_{rz}$
   - Uses SVD and column compression on the (deflated) pencil $G^{(k)} - \lambda H^{(k)}$, $k = 0, 1, \ldots$
   - Column compress $G^{(k)}$ using $\mathrm{SVD}(G^{(k)}) = (*, \Sigma_A, V_A)$:

     $s_k =$ dim column nullspace of $G^{(k)} = \#(\sigma(G^{(k)}) < tolG)$ (insist on gap)

The Guptri Algorithm

Seven steps in total:

1. Compute the RZ-staircase form (RZ-algorithm):
   Right singular structures and zero eigenvalues in $A_{rz} - \lambda B_{rz}$
2. Separate right singular part from regular part:
   Obtain $A_r - \lambda B_r$ in staircase form
   - Apply RZ-algorithm on $B_{rz} - \mu A_{rz}$
   - Insist on the same right singular structure as in Step 1

THE GUPTRI ALGORITHM

Seven steps in total:

1. Compute the RZ-staircase form (RZ-algorithm):
   Right singular structures and zero eigenvalues in $A_{rz} - \lambda B_{rz}$
2. Separate right singular part from regular part:
   Obtain $A_r - \lambda B_r$ in staircase form
3. Obtain $A_z - \lambda B_z$ in staircase form
   - Apply RZ-algorithm on $A_z - \lambda B_z$
   - Insist on the same regular structure as in Step 1

The Guptri Algorithm

Seven steps in total:

1. Compute the RZ-staircase form (RZ-algorithm):
   Right singular structures and zero eigenvalues in $A_{rz} - \lambda B_{rz}$
2. Separate right singular part from regular part:
   Obtain $A_r - \lambda B_r$ in staircase form
3. Obtain $A_z - \lambda B_z$ in staircase form

4-6. Apply the LI-algorithm (from lower-right corner):
Obtain left singular structures in $A_l - \lambda B_l$ and infinite eigenvalues in $A_i - \lambda B_i$
   - Work on $B - \mu A$ using row compression
   - In total 3 steps

The Guptri Algorithm

Seven steps in total:

1. Compute the RZ-staircase form (RZ-algorithm):
   Right singular structures and zero eigenvalues in $A_{rz} - \lambda B_{rz}$
2. Separate right singular part from regular part:
   Obtain $A_r - \lambda B_r$ in staircase form
3. Obtain $A_z - \lambda B_z$ in staircase form

4-6. Apply the LI-algorithm (from lower-right corner):
   Obtain left singular structures in $A_l - \lambda B_l$ and
   infinite eigenvalues in $A_i - \lambda B_i$

7. Apply RZ-algorithm on the remaining square regular pencil:
   Finite non-zero eigenvalues in $A_f - \lambda B_f$
   - Includes clustering of the eigenvalues

GUPTRI ALGORITHM ILLUSTRATED

Initial matrix:

$$G - \lambda H \qquad =$$

$$\begin{bmatrix} * \end{bmatrix} - \lambda \begin{bmatrix} * \end{bmatrix}$$

Guptri Algorithm Illustrated

Step 1:

$$P_{rz}^{H}(G - \lambda H)Q_{rz} =$$

$$\begin{bmatrix} A_{rz} & * \\ 0 & * \end{bmatrix} - \lambda \begin{bmatrix} B_{rz} & * \\ 0 & * \end{bmatrix}$$

GUPTRI ALGORITHM ILLUSTRATED

Steps 2–3:

$$P_{r-z}^{H} P_{rz}^{H} (G - \lambda H) Q_{rz} Q_{r-z} =$$

$$\begin{bmatrix} A_r & * & * \\ 0 & A_z & * \\ 0 & 0 & * \end{bmatrix} - \lambda \begin{bmatrix} B_r & * & * \\ 0 & B_z & * \\ 0 & 0 & * \end{bmatrix}$$

GUPTRI ALGORITHM ILLUSTRATED

Step 4:

$$P_{li}^H P_{r-z}^H P_{rz}^H (G - \lambda H) Q_{rz} Q_{r-z} Q_{li} \quad =$$

$$\begin{bmatrix} A_r & * & * & * \\ 0 & A_z & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & A_{li} \end{bmatrix} - \lambda \begin{bmatrix} B_r & * & * & * \\ 0 & B_z & * & * \\ 0 & 0 & * & * \\ 0 & 0 & 0 & B_{li} \end{bmatrix}$$

GUPTRI ALGORITHM ILLUSTRATED

Steps 5–6:

$$P_{l-i}^H P_{li}^H P_{r-z}^H P_{rz}^H (G - \lambda H) Q_{rz} Q_{r-z} Q_{li} Q_{l-i} \quad =$$

$$\begin{bmatrix} A_r & * & * & * & * \\ 0 & A_z & * & * & * \\ 0 & 0 & * & * & * \\ 0 & 0 & 0 & A_i & * \\ 0 & 0 & 0 & 0 & A_l \end{bmatrix} - \lambda \begin{bmatrix} B_r & * & * & * & * \\ 0 & B_z & * & * & * \\ 0 & 0 & * & * & * \\ 0 & 0 & 0 & B_i & * \\ 0 & 0 & 0 & 0 & B_l \end{bmatrix}$$

Guptri Algorithm Illustrated

Step 7:

$$P_f^H P_{l-i}^H P_{li}^H P_{r-z}^H P_{rz}^H (G - \lambda H) Q_{rz} Q_{r-z} Q_{li} Q_{l-i} Q_f =$$

$$\begin{bmatrix} A_r & * & * & * & * \\ 0 & A_z & * & * & * \\ 0 & 0 & A_f & * & * \\ 0 & 0 & 0 & A_i & * \\ 0 & 0 & 0 & 0 & A_l \end{bmatrix} - \lambda \begin{bmatrix} B_r & * & * & * & * \\ 0 & B_z & * & * & * \\ 0 & 0 & B_f & * & * \\ 0 & 0 & 0 & B_i & * \\ 0 & 0 & 0 & 0 & B_l \end{bmatrix}$$

where a matrix pair (A_x, B_x) is non-existing (0×0) for an absent invariant

CLUSTERING OF EIGENVALUES - MOTIVATION

Let A be a 5×5 matrix in JCF

$$A = \begin{bmatrix} 3 & 1 & 0 & 0 & 0 \\ 0 & 3 & 1 & 0 & 0 \\ 0 & 0 & 3 & 1 & 0 \\ 0 & 0 & 0 & 3 & 1 \\ \alpha & 0 & 0 & 0 & 3 \end{bmatrix}$$

If $\alpha = 0$, A has one eigenvalue $\mu = 3$ with multiplicity 5

CLUSTERING OF EIGENVALUES - MOTIVATION

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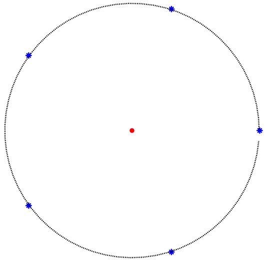
If $\alpha = 0$, A has one eigenvalue $\mu = 3$ with multiplicity 5

If $\alpha = 10^{-14}$, A has 5 eigenvalues at the distance $\sqrt[n]{\alpha} = \sqrt[5]{10^{-14}}$ from μ



Clustering of Eigenvalues

Hierarchical cluster methods:

- Using Gershgorin circles to cluster the eigenvalues starting with the eigenvalue closest to a goal
  [Gershgorin'31, Kågström-Wiberg'92]

CLUSTERING OF EIGENVALUES

Hierarchical cluster methods:

- Using Gershgorin circles to cluster the eigenvalues starting with the eigenvalue closest to a goal [Gershgorin'31, Kågström-Wiberg'92]
- Using a distance function $D(X,Y) \leq \delta_{max}$ where X and Y are different clusters of eigenvalues, and where a minimum number P_{min} of clusters can be guaranteed [Kintzel'03]

  Possible functions $D(X,Y)$ are:

min	$\min(\text{abs}(X(i) - Y(j)))$, for all i, j
max	$\max(\text{abs}(X(i) - Y(j)))$, for all i, j
average	$1/(rs)(\sum^{r,s} \text{abs}(X(i) - Y(j)))$
mean	$\text{abs}(\sum(X)/r - \sum(Y)/s)$
variance	$\text{var}(X \cup Y)$

  where r and s are #eigenvalues in each cluster

Matrix Canonical Structure (MCS) Toolbox

The MCS Toolbox – A Matlab toolbox for computing and representing canonical structure information

Includes data type objects for

- Canonical Structure Objects
- Canonical Block Objects

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StratiGraph – Software tool for stratification of closure hierarchies

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Available canonical structure objects:

`pstruct`	Matrix pencils under strict equivalence
`mstruct`	Matrices under similarity
`mpstruct`	Matrix polynomials under strict equivalence of the corresponding Fiedler linearization
`ssstruct`	State-space system pencils under feedback-injection equivalence
`cmstruct`	Matrices under congruence
`scmstruct`	Matrices under *congruence
`spstruct`	Symmetric matrix pencils under congruence
`sspstruct`	Skew-symmetric matrix pencils under congruence

Canonical Structure Objects

Methods in canonical structure objects for

- accessing and changing the canonical structure
  - `get`, `set`, `compare`, `copy`, etc.
- obtaining sizes and codimension of the matrix representation
  - `size`, `numblk`, `codim`, etc.
- obtaining the corresponding canonical form in matrix form
  - `jnf`, `kcf`, `bcf`, etc.

Canonical Block Objects

Canonical Block Objects are data type objects for representing canonical blocks

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Available canonical block objects:

- `fjblock` Jordan block for a finite eigenvalues
- `zjblock` Jordan block for the zero eigenvalue
- `ijblock` Jordan block for the infinite eigenvalue
- `rsblock` Right singular block
- `lsblock` Left singular block
- ... And corresponding block objects for (skew-)symmetric matrix pencils and matrices under congruence and *congruence

EXAMPLE - CANONICAL STRUCTURE OBJECT

Create a matrix pencil object with KCF $L_2 \oplus L_0^T \oplus 2J_1(3+i2)$

```
>> pstr = pstruct('rsblock',2,'lsblock',0,'fjblock',{[1 1],3+2i})
pstr =
     R2 + L0 + 2*J1(3 + 2i)
```

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Get the size of the matrix pencil

```
>> pstr.size
ans =
     5     5
```

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Get the size of the matrix pencil

```
>> pstr.size
ans =
     5     5
```

Get the block object for the Jordan blocks

```
>> fjblk = pstr.get('fjblock','object');
>> fjblk{1}
ans =
     2*J1(3.0000 + 2.0000i)
```

Example - Canonical Structure Object

Recall pstr has the KCF $L_2 \oplus L_0^T \oplus 2J_1(3+i2)$

```
>> pstr = pstruct('rsblock',2,'lsblock',0,'fjblock',{[1 1],3+2i})
pstr =
     R2 + L0 + 2*J1(3 + 2i)
```

Get the KCF in matrix form

```
>> [G,H] = pstr.kcf
G =                                                 H =
     0+0i   1+0i   0+0i   0+0i   0+0i                    1     0     0     0     0
     0+0i   0+0i   1+0i   0+0i   0+0i                    0     1     0     0     0
     0+0i   0+0i   0+0i   0+0i   0+0i                    0     0     0     0     0
     0+0i   0+0i   0+0i   3+2i   0+0i                    0     0     0     1     0
     0+0i   0+0i   0+0i   0+0i   3+2i                    0     0     0     0     1
```

Matrix Canonical Structure (MCS) Toolbox

Numerical routines for computing

- the *staircase form* (the Guptri form) of matrices and matrix pencils
  - `mguptri` and `pguptri`

MATRIX CANONICAL STRUCTURE (MCS) TOOLBOX

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- the *staircase form* (the Guptri form) of matrices and matrix pencils
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- a matrix representation of the *tangent space* of an orbit
- the *codimension*, either from matrices or the canonical structure

Matrix Canonical Structure (MCS) Toolbox

Numerical routines for computing

- the *staircase form* (the Guptri form) of matrices and matrix pencils
  - `mguptri` and `pguptri`
- a matrix representation of the *tangent space* of an orbit
- the *codimension*, either from matrices or the canonical structure

Also includes

- a gallery of sample matrices and matrix pencils for staircase computations

THE GUPTRI FORM FOR MATRIX PENCILS

```
pstr = pguptri(G,H)
```

compute the canonical structure information of $G - \lambda H$
where `pstr` is a matrix pencil object

The Guptri Form For Matrix Pencils

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pstr = pguptri(G,H)
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compute the canonical structure information of $G - \lambda H$
where `pstr` is a matrix pencil object

Returning the Guptri staircase form:

```
[A,B,P,Q,dG,dH,pstr] = pguptri(G,H)
```

where P and Q are unitary and A and B are in generalized upper triangular form (Guptri staircase form), such that

$$P(A - \lambda B)Q^H = (G + dG) - \lambda(H + dH)$$

The Guptri Form For Matrix Pencils

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```

compute the canonical structure information of $G - \lambda H$
where `pstr` is a matrix pencil object

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where P and Q are unitary and A and B are in generalized upper triangular form (Guptri staircase form), such that

$$P(A - \lambda B)Q^H = (G + dG) - \lambda(H + dH)$$

$\|(dG, dH)\|_F$ is an upper bound on the distance from $G - \lambda H$ to a matrix pencil with the computed Kronecker structure

UPPER BOUND ON ERROR

If the small singular values $< tol$ are kept in the deflation

$$\|(dG, dH)\| \approx \|(G, H)\| * eps$$

Otherwise

$$\|(dG, dH)\| \approx \|(G, H)\| * tol$$

Specifying Regular Block Order

The block order of the regular part

$$A_{reg} - \lambda B_{reg} = \begin{bmatrix} A_z & * & * \\ 0 & A_f & * \\ 0 & 0 & A_i \end{bmatrix} - \lambda \begin{bmatrix} B_z & * & * \\ 0 & B_f & * \\ 0 & 0 & B_i \end{bmatrix}$$

can be changed

- Reordering of the sub-pencils are done by solving a generalized Sylvester equation

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can be changed

- Reordering of the sub-pencils are done by solving a generalized Sylvester equation

Specified with an optional parameter to `guptri`

'zfi'	Zero-Finite-Infinite	**'fiz'**	Finite-Infinite-Zero
'zif'	Zero-Infinite-Finite	**'ifz'**	Infinite-Finite-Zero
'fzi'	Finite-Zero-Infinite	**'izf'**	Infinite-Zero-Finite

CLUSTERING OF EIGENVALUES

Eigenvalue clustering method is specified with (default 'var')

'norm'	Cluster within a norm distance
'gersh'	Cluster using Gershgorin circles
'min'	Cluster using a hierarchical clustering algorithm
'max'	where the clustering is determined by the specified
'avg'	distance function $D(X, Y)$
'mean'	
'var'	

Pguptri - Example

Load the sample matrix pencil 'boley2' [Boley'90] with KCF L_3

```
>> [G,H] = pggallery('boley2');
```

where

$$G - \lambda H = \begin{bmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} - \lambda \begin{bmatrix} 1 & -149 & 537 & -27 \\ 1 & -50 & 180 & -9 \\ 1 & -154 & 546 & -25 \end{bmatrix}$$

PGUPTRI - EXAMPLE

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Results from pguptri ($tol = 10^{-12}$ and $gap = 1000$)

```
>> [A,B,pstr] = pguptri(G,H)
```

```
A =
   -1.0000 -0.0046   0.0008 -0.0004
         0  0.1966   0.8556 -0.4789
         0       0   0.4884  0.8726

B =
     0 -146.8650 -492.6903   635.9285
     0         0    2.3417    -1.5560
     0         0         0     3.7346

pstr =
     R3
```

Pguptri - Problematic Example

The 3×4 sample matrix pencil 'test1' with KCF $L_0 \oplus L_1 \oplus L_2^T \oplus J_1(10^{-4}i)$ has ill-conditioned matrices G and H, $O(\text{cond}(G)) = O(\text{cond}(H)) = 10^{16}$

pguptri fails:

```
>> [A,B,P,Q,dG,dH,pstr] = pguptri(G,H);
```

```
Warning: The rank is increased by 1 with a total perturbation
  of 1.000e-12.
Warning: Failed to separate the right singular and the zero
  eigenvalue blocks.
Warning: Returned order RZ-F-I-L, where RZ is not separated.
```

returns the KCF $L_0 \oplus J_3(0)$

Pguptri - Problematic Example

The 3×4 sample matrix pencil 'test1' with KCF $L_0 \oplus L_1 \oplus L_2^T \oplus J_1(10^{-4}i)$ has ill-conditioned matrices G and H, $O(\text{cond}(G)) = O(\text{cond}(H)) = 10^{16}$

However,
the correct KCF is computed if the matrices are swapped in the computation

```
>> pstr = pguptri(G,H,'ifz')
pstr =
     R1 + R0 + L0 + J1(-3.7259e-17 + 1.0000e-04i)
```

Download and Contact

MCS Toolbox v0.6 and StratiGraph v4.0 can be downloaded from
`https://www.umu.se/en/stratigraph-mcs/`

MCS Toolbox v0.7 including pguptri is available after summer
or send an email to stefanj@cs.umu.se