StratiGraph and the Matrix Canonical Structure Toolbox

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TOOLS FOR COMPUTING AND ANALYZING CANONICAL STRUCTURE INFORMATION

Study of canonical forms for

▶ Matrices, matrices of bilinear forms
▶ Matrix pencils $G - \lambda H$ (singular, regular, (skew-)symmetric)
▶ System pencils associated with generalized state-space systems

$$\dot{Ex} = Ax + Bu, \quad y = Cx + Du$$

▶ Linearizations of matrix polynomials $P(s) = P_d s^d + \cdots + P_1 s + P_0$

Motivation

▶ Determining the dynamics and system characteristics (like poles, zeros, controllability)
  ▷ Understand how small changes in the system matrices, e.g., due to round-off errors or external disturbances, may affect the system characteristics

Ill-posed problems!

▶ Small perturbations of data matrices can drastically change the computed canonical structure information

Qualitative information from orbit and bundle Stratifications

▶ Provides information for a deeper understanding how system transitions can take place under small perturbations
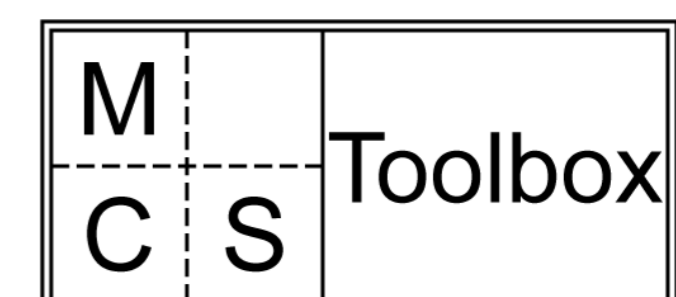▶ Reveals the closure hierarchy of orbits and bundles

$\Rightarrow$ *StratiGraph*

Quantitative information

▶ Canonical structure information
▶ Staircase forms: reveals the canonical structure information
▶ Upper and lower bounds to nearby canonical structures
▶ Distance to uncontrollability

$\Rightarrow$ *Matrix Canonical Structure Toolbox* for Matlab

StratiGraph

StratiGraph* is a Java-based tool to compute, view, and investigate qualitative information on the relation between different canonical structures of an input setup.

The stratification reveals the closure hierarchy of obits (or bundles) of the setup, where all, e.g., matrices, in an orbit have the same canonical form.

Supported problem setups (new in coming version 3.2)

▶ Matrices under similarity
▶ Matrix pencils $G - \lambda H$ under strict equivalence
▶ System pencils

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} - s \begin{bmatrix} E & 0 \\ 0 & 0 \end{bmatrix}, \quad \det(E) \neq 0$$

  ▷ Controllability pairs (A,B) under feedback equivalence
  ▷ Observability pairs (A,C) under injection equivalence
▶ Matrix polynomial Fiedler linearizations
  ▷ Matrix polynomials of full normal rank

Other features (new in coming version 3.2)

▶ Export/import setups to/from Matlab
▶ Expand whole or part of the graph (e.g., around a canonical structure of interest)
▶ Nodes can be marked or hidden by filters on the canonical structure
▶ The canonical structure information can be shown in different notations
▶ Expandable with new setups and extensions using plug-ins
▶ OS independent

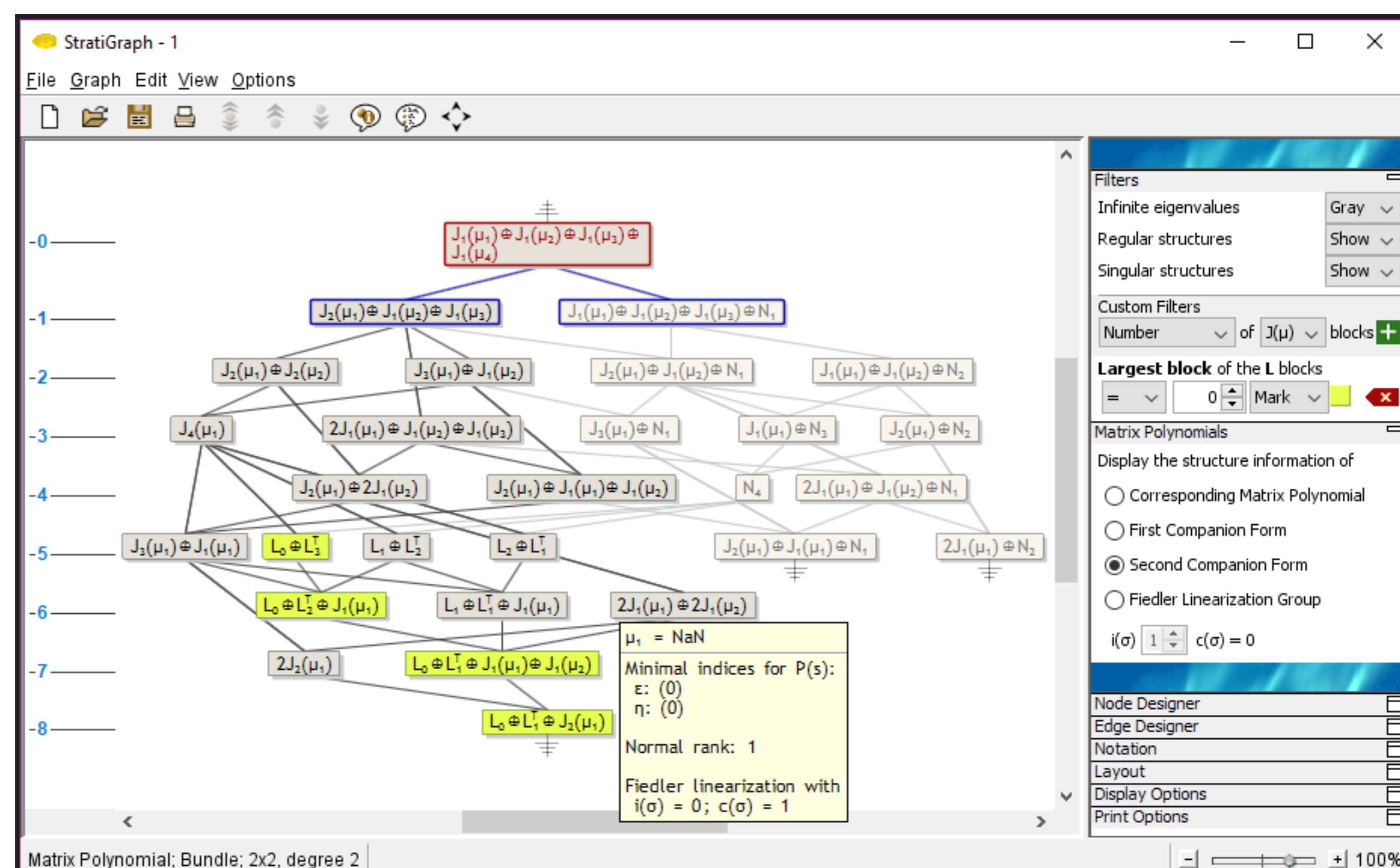* StratiGraph is an acronym for *Strati*fication *Graph*s.

Figure: StratiGraph: Closure hierarchy graph—nodes are orbits, edges show covering relations (nearest neighbors) between bundles. The top-most node is the most generic bundle. The bottom node is the most degenerate.

MATRIX CANONICAL STRUCTURE TOOLBOX

The Matrix Canonical Structure (MCS) Toolbox is a Matlab toolbox including both computational routines and a framework for representing canonical structure objects.

The canonical structure is defined by the regular structures (finite and infinite eigenvalues together with their geometric/algebraic multiplicities) and the singular structures (right/left minimal indices), depending on which are applicable.

Data type objects exist for representing canonical structures of

▶ Matrices (under similarity, congruence, and *congruence)
▶ Matrix pencils (under strict equivalence, and symmetric and skew-symmetric pencils under congruence)
▶ System pencils (under feedback-injection equivalence)
▶ Matrix polynomials (under strict equivalence of the corresponding Fiedler linearization)

Numerical routines exist for computing

▶ a matrix representation of the Tangent space of an orbit
▶ the Codimension, either from matrices or the canonical structure information
▶ the Staircase form (the Guptri form) of matrices and matrix pencils (*in coming release*)

POSSIBLE FUTURE EXTENSIONS

▶ Matrix polynomials with structure
▶ Generalized state-space systems (descriptor systems)
▶ Upper and lower bounds of distances between canonical structures
▶ Staircase forms for more supported canonical structure objects
▶ and more...

FURTHER INFORMATION

For references, contact information, and download see

http://www.cs.umu.se/english/research/groups/matrix-computations/stratigraph/

EXTERNAL FUNDING